Interactions between water waves and floating structures, a numerical method

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Abstract

We propose a FEM-based numerical approach for wave–floating-structure interactions. The evolution of the 2D fluid is governed by the incompressible free-surface Navier-Stokes equations. The motion of the rigid body is prescribed by Newton's equations, accounting for both the gravitational and buoyancy forces.

Keywords: Wave–structure interaction, Finite Element Method, ALE, Navier-Stokes equations.

1 Floating body and buoyancy

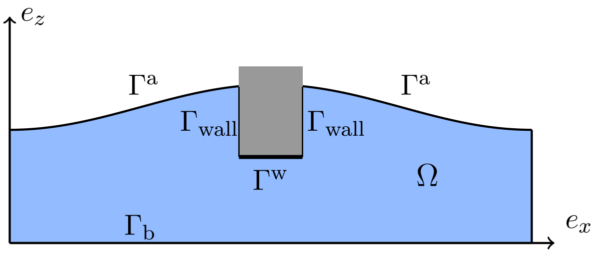

Figure 1: Schematic representation of the configuration.

Consider a freely-floating rigid body in a viscous two-dimensional fluid of density ρ and viscosity ν. The fluid domain is denoted Ω. Its boundary is decomposed as $\partial\Omega = \Gamma^{\mathrm{a}} \cup \Gamma^{\mathrm{w}} \cup \Gamma_{\mathrm{b}} \cup \Gamma_{\mathrm{wall}}$ (Fig. 1). Due to the displacement of both the free surface and the floating body, the quantities Ω, Γ^{a}, Γ^{w} and Γ_{wall} are, in fact, time-dependent. For the sake of simplicity and brevity, we shall only consider the case of a rectangular body moving in the mere vertical direction.

The distance δ between the center of mass and the body equilibrium position is given by the Newton equation

$$m\ddot{\delta} + \rho g|\Gamma^{\mathrm{w}}|\delta = \int_{\Gamma^{\mathrm{w}}} (\Pi - 2\nu\partial_z u_z)$$

where the fluid velocity u and the hydrodynamical pressure $\Pi = P - P_{\mathrm{atm}} + \rho g z$ are computed through the incompressible free-surface Navier-Stokes equations on Γ^{a}, homogeneous Dirichlet on $\Gamma^{\mathrm{w}} \cup \Gamma_{\mathrm{b}}$ and $u_{|\Gamma^{\mathrm{w}}} = \dot{\delta} e_z$ at the wetted line Γ^{w}. The equation on the vertical displacement δ is in fact a combination of Newton's equations, accounting for both the gravitational and the stress exerted on the solid by the fluid (including the viscosity), and the Archimedes equilibrium equation.

2 Added mass effect

When a partially submerged structure accelerates in a fluid, it does not move alone: it sets a portion of the surrounding fluid into motion too. The added mass corresponds precisely to the inertial component of this supplementary fluid motion. This inertial contribution is contained in the pressure forces acting on the body's wetted interface. Let us first consider an ideal, incompressible, and irrotational fluid. In this setting, the velocity field is the gradient of a scalar harmonic potential ϕ. This potential depends linearly on the velocity of the solid. More precisely $\phi(t, x) = \dot{\delta}(t)\phi(t, x) + \phi_{\mathrm{fluid}}(t, x)$ where ϕ is the harmonic potential which satisfies at the boundaries:

$$(\partial_n \phi)_{|\Gamma^{\mathrm{w}}} = 1, \quad (\partial_n \phi)_{|\Gamma_{\mathrm{b}} \cup \Gamma_{\mathrm{wall}}} = 0, \quad \phi_{|\Gamma^{\mathrm{a}}} = 0.$$

Note that since the fluid domain moves, this harmonic potential ϕ consequently depends on time.

Within the framework of irrotational fluids, the pressure can be computed from the potential through Bernoulli's equation, relating the evolution of ϕ to the velocity's squared Euclidean norm and the pressure. When the latter is integrated over the surface of the body to evaluate the hydrodynamic force, the term involving the time derivative of the potential produces a contribution proportional to the acceleration of the solid. We can interpret this inertial contribution as an added mass. Within the Navier-Stokes framework, the velocity field can be decomposed into the gradient of a potential responsible for the added mass effects, plus a non-inertial remainder that can no longer be expressed as a gradient due to viscous effects. Then the hydrodynamical pressure is decomposed into an added-mass term and a non-inertial hydrodynamical pressure $\Pi = -\rho\ddot{\delta}\varphi + \widetilde{\Pi}$. Thus, the Newton equation becomes

$$(m + m_a)\ddot{\delta} + \rho g|\Gamma^{\mathrm{w}}|\delta = \widetilde{f}_{\mathrm{hyd}} := \int_{\Gamma^{\mathrm{w}}} (\widetilde{\Pi} - \nu\partial_z u_z)$$

where the added mass is given by

$$m_a := \int_\Omega \rho|\nabla\varphi|^2.$$

The evolution of the fluid's quantities $(u, \widetilde{\Pi})$ is achieved through the Navier-Stokes equations:

$$\begin{cases} \rho D_t u + \nabla\widetilde{\Pi} - \rho\nu\Delta u = a_\delta\nabla\varphi & \text{in } \Omega(t), \\ \nabla\cdot u = 0 & \text{in } \Omega(t), \\ \widetilde{\Pi} n + 2\nu D[u]n = a_\delta\varphi n + \rho g z n & \text{on } \Gamma^{\mathrm{a}}(t), \\ u = 0 & \text{on } \Gamma_{\mathrm{b}} \cup \Gamma_{\mathrm{wall}}(t), \\ u = \dot{\delta} e_z & \text{on } \Gamma^{\mathrm{w}}(t), \end{cases}$$

where

$$a_\delta := \rho\frac{\widetilde{f}_{\mathrm{hyd}} - \rho g|\Gamma^{\mathrm{w}}|\delta}{m + m_a}, \qquad D_t := \partial_t + (u\nabla).$$

We denoted as $D[u]$ is the symmetric part of the gradient tensor and n the unit vector perpendicular to the free surface.

3 Numerical method

Numerical approximation of the solution to this problem is achieved extending the scheme introduced in [3] to handle the movements of the solid body. At each time iteration, the harmonic potential ϕ, the velocity u and the pressure p are approximated using the Finite Element Method, implemented using the FreeFEM library [1]. Taylor-Hood $(\mathbb{P}^2, \mathbb{P}^1)$ elements are used for the Navier-Stokes solver. Advection of the free-surface $\Gamma^{\mathrm{a}}(t)$ and of the wetted boundary $\Gamma^{\mathrm{w}}(t) \cup \Gamma_{\mathrm{wall}}(t)$ is achieved transporting the whole triangulation with the fluid (the ALE method of [2]). This ensures an accurate depiction of the movement of both the free surface and the floating solid body. The fluid's pressure and velocity, needed for Newton's equation, are first computed during a predictor step. The solid's velocity $\dot{\delta}$ is then computed from these first estimates. From this, the fluid's pressure and velocity are recomputed (corrector step) and then used to advect the mesh.

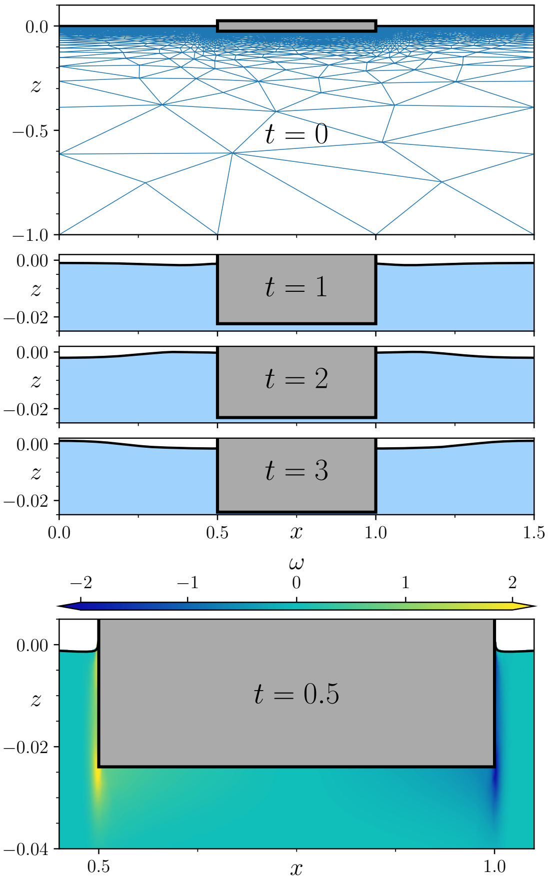

Figure 2: Simulation of an immersed solid moving upward due to buoyancy: initial mesh (top), free surface evolution (middle) and vorticity ω, reminiscent of the separation observed in [4] around fixed obstacles (bottom).

Data accessibility and reproducibility

The FreeFEM scripts along with the post-processing python notebook can be found `here`.

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References

[1] Hecht, F. (**2012**). *J. Numer. Math.* **20**:3-4, 251–265, `https://freefem.org/`.

[2] C.W. Hirt, A.A. Amsden, J.L. Cook (**1974**). *J. Comp. Phys.* **14**:3, 227-253.

[3] A. Riquier, E. Dormy (**2024**). In: *J. Fluid Mech.* **984**, R5.

[4] A. Riquier, E. Dormy (**2026**). In: *Proc. Roy. Soc. A* **482**(2329):20250772.